UNSTEADY CONJUGATE HEAT EXCHANGE OF OIL-GAS PIPE LINE IN SOIL

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Introduction

The problem of conjugate heat exchange of main gas-oil pipe lines is of special importance for pipelines laying in the permafrost regions, because the energy saving, reliability and ecology problems ought to be solved. In this work, this complex mathematical task without allowance for a movable thawing bound was numerically simulated.

The calculating results of conjugate unstable heat exchange of a "hot" product-pipeline with a uniform soil were presented at the following assumptions:

- The flow regime in a tube is turbulent and unsteady, the liquid is incompressed, and the physical properties are constant.
- Heat exchange along the pipeline axis is negligibly small in comparison with the heat exchange normal to the pipeline axis.
- The soil in which the product-pipeline is embedded is a uniform massif with known thermal-physical properties.
- The pipe wall and its insulation is negligible thin, and their thermal resistance is modeled by a certain coefficient in the boundary condition at the pipe and soil junction.

The task for the pipe at a flow hydraulic stabilizing comes to solution of the energy unstationary equation:

$$\frac{\partial T}{\partial \tau} + W\frac{\partial T}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\,(a + a_T)\frac{\partial T}{\partial r}\right], \qquad (1)$$

where T is a liquid temperature; τ is a time; a, a_T are the coefficients of a temperature conductivity of the laminar and turbulent flow regimes accordingly; W is a velocity axis component;

Equation (1) describes the unsteady heat exchange of liquid and the pipeline wall at the turbulent flow regime, and the velocity field is described by the Reichard dependence [1] and [2]. Equation (1) is a hyperbolic-parabolic (ultra-parabolic) that are considered at difference scheme construction.

The temperature distribution in soil at undersoil pipe laying taking into account a day surface is described by a non-stationary, two-dimensional equation of the thermal conductivity. In such case, the longitudinal heat flows are neglected in comparison with the transversal ones. Thus, a three-dimensional problem of the thermal conductivity is split in a great number of two-dimensional problems connected by boundary conditions on a pipe wall:

$$\frac{\partial t}{\partial \tau} = a_{\mathrm{s}}\left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2}\right), \qquad (2)$$

t is a soil temperature; a_{s} is the coefficient of the soil thermal conductivity; x , y are the co-ordinates of the cross section; x is directed along the soil bound and the day surface from a symmetry plane; axis y – lies in the symmetry plane and directed into the soil from the day surface [3].

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Abstract

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The thermal conductivity along the axis z in the soil is negligibly small.

The coordinates of the pipe center (*0*, *H*), where *H* is a depth of the pipe laying, and the equation of the pipe circle has a form: $x^2+(y-H)^2=r_0^2$.

An area of the equation integration (2):

$$\left\{0<\tau<\tau_k;\ -\infty<x<\infty;\ 0<y<\infty;\ x^2+(y-H)^2>r_0^2\right\},$$

with the bound conditions on the day surface:

$$\lambda_{\text{гр}}\frac{\partial t}{\partial y}-\alpha_{\text{air}}\,(t-t_{\text{air}})=0 \text{ at } y=0; \tag{3}$$

at the bound of the pipe and soil the conjugate conditions are written, that are equalities of the thermal flows and temperatures in the form:

$$q_{\text{s}}=q_{\text{liq}},\ t_{\text{s}}=\delta\ T_{\text{liq}}, \tag{4}$$

where λ_{s} is the coefficient of the soil thermal conductivity; α_{air} is the coefficient of a heat transfer from the soil to air; r_0 − is a radius of a thin-wall pipe. Because the area, where the solution is determined, is symmetrical relatively to the axis $0y$, the condition

$$\lambda_{\text{s}}\frac{\partial t}{\partial x}=0 \text{ at } 0\le y\le H-r_0;\ H+r_0\le y<\infty \tag{5}$$

is added at the bound $x=0$.

The endless integration area in the soil is transformed into the circle one using a conform transformation [3]: ($\mu=\ln R$)

$$\omega=\frac{\Pi-c}{\Pi+c},\ \Pi=y+ix,\ c=\sqrt{H^2-r_0^2},\text{ where } \omega=\exp(\mu+i\varphi).$$

The equation (2) in new dimensionless coordinates has a form:

$$\frac{\partial U}{\partial F_0}=a_w(\mu,\varphi)\left(\frac{\partial^2 U}{\partial\mu^2}+\frac{\partial^2 U}{\partial\varphi^2}\right), \tag{6}$$

Boundary conditions (3) – (5) in variables μ,φ in the surface ω take the form:

$$(1-\cos\varphi)\frac{\partial U}{\partial\mu}+Bi\ U=0 \text{ at } \mu=0, \tag{7}$$

The symmetry conditions relatively to the axis $0y$:

$$\frac{\partial U}{\partial\varphi}=0 \text{ at } \varphi=0 \text{ and } \varphi=\pi. \tag{8}$$

Symbols μ,φ are the polar coefficients in a complex plane ω;

$$Bi=\frac{c\ \alpha_{\text{air}}}{\lambda_{\text{s}}},\ F_0=\frac{\tau\ a}{r_0^2},\ \mu_0=\ln\left(\frac{r_0}{H+c}\right)<0.$$

$$U=\frac{t}{t_{\text{air}}}-1,\ a_w(\mu,\varphi)=\frac{a_{\text{s}}}{a_{\text{liq}}}\left(\frac{r_0}{c}\right)^2(ch\mu-\cos\varphi)^2,$$

To construct a numerical algorithms it is necessary to express the derivatives on normal both at the day surface boundary and at the pipeline plane by new variables.

In the result, derivatives on variables x, y will be written in the form:

$$\frac{\partial}{\partial y}=\frac{1}{c}\left[(ch\mu\cos\varphi-1)\frac{\partial}{\partial\mu}+sh\mu\sin\varphi\frac{\partial}{\partial\varphi}\right],$$
$$\frac{\partial}{\partial x}=\frac{1}{c}\left[-sh\mu\sin\varphi\frac{\partial}{\partial\mu}+0.5\,(ch\mu\cos\varphi-1)\frac{\partial}{\partial\varphi}\right]. \tag{9}$$

On the day plane at $\mu=0$: $\quad \frac{\partial}{\partial y_{y=0}}=\frac{1}{c}\left(\cos\varphi-1\right)\frac{\partial}{\partial\mu}$.

The derivative on normal to the pipeline surface coincides with the derivative in radius of a polar coordinate system in the physical surface connected with the pipeline axis:

$$\frac{\partial}{\partial n}_{r=r_0}=\frac{1}{c}\left(\frac{H}{c}-\cos\varphi\right)\frac{\partial}{\partial\mu}_{\mu=\mu_0}; \tag{10}$$

These expressions for derivatives are necessary to write the conjugate conditions at a contact pipe surface with the soil (4) in new variables.

Transformation of thermal exchange equation (1) at the flow turbulent regime in the pipeline [1] .

$$\frac{\partial\Theta}{\partial F_0}+V\frac{\partial\Theta}{\partial Z}=\frac{(\rho+\varepsilon)^3}{\varepsilon(1+\varepsilon)^2}\frac{1}{\rho}\frac{\partial}{\partial\rho}\left[\gamma(\rho)\frac{(\rho+\varepsilon)}{\varepsilon}\rho\frac{\partial\Theta}{\partial\rho}\right], \tag{11}$$

where $\Theta=1-\frac{T}{T_e}$; $V=\frac{W}{W_{\max}}$; $Z=\frac{2z}{r_0\ P_e}\frac{W_{\mathrm{m}}}{W_{\max}}$; $\gamma(\rho)=\frac{a\ +\ a_T}{a}\ =1+\frac{P_r}{P_T}\frac{\nu_T}{\nu}$; is a dimensionless transformed (deformated) pipe radius; $\varepsilon=\mathrm{const}$ is a stretching parameter of a pipe physical radius which is connected with the Reynolds dependence: $\varepsilon=1/\left(0.005\ \mathrm{Re}^{3/4}-1\right)$. Using of the physical radius transformation is necessary to compensate a large near-wall profiles gradient of a velocity and temperature at the turbulent flow. W_{m} , $W_{\max}$ are mean and maximum values on the flow velocity axis. $P_e=2W_{\mathrm{m}}r_0/a$, P_r , P_T are the Peckle and Prandtl numbers laminar and turbulent

The conjugate conditions (4) are written in the form:

$$\frac{\lambda_{\mathrm{liq}}T_e}{r_0}\left(\frac{1\ +\ \varepsilon}{\varepsilon}\right)\frac{\partial\Theta}{\partial\rho_{\rho=1}}=-\frac{\lambda_{\mathrm{s}}t_{\mathrm{air}}}{c}\left(\frac{H}{r_0}-\cos\varphi\right)\frac{\partial U}{\partial\mu_{\mu=\mu_0}},$$
$$\delta\ (1+U_{11})\ t_{\mathrm{air}}=(1-\Theta_L)\ T_e\ \text{ at }\ \mu=\mu_0\ . \tag{12}$$

An attempt to construct an algorithm of a direct solution of the conjugate problem of the pipeline thermal exchange with the soil was made. To pursue this aim, a method of variable directions or the splitting method on spatial variables of N.N. Yanenko [4] was applied to integrate the equations (6). Two-dimensional equation is splitted into two one-dimensional equations:

$$\frac{\partial U_1}{\partial F_0}=a_w\frac{\partial^2 U_1}{\partial\mu^2},\quad \frac{\partial U_2}{\partial F_0}=a_w\frac{\partial^2 U_2}{\partial\varphi^2}\ . \tag{13}$$

Integrating areas sections $\left(\mu_0 \le \mu \le 0\right)$ and $\left(0 \le \varphi \le \pi\right)$ are separated into N–1 and M–1 accordingly with steps: $\left(\Delta F_0{}^{j} = F_0^{j+1} - F_0^{j}\right), \left(\Delta\mu_n = \mu_{n+1} - \mu_n\ \right) and \left(\Delta\varphi_m = \varphi_{m+1} - \varphi_m\right)$. The system is integrated at every step on time. Indexes m, n run the values (n = 2, 3, 4,…, N–2, N–1), (m =2, 3, 4,…, M–2, M–1) accordingly. The difference equations for system (13) can be presented in a three-diagonal form:

$$A_{1n}\, U_{1\,n+1}^{j+1} + B_{1n}\, U_{1\,n}^{j+1} + C_{1n}\, U_{1\,n-1}^{j+1} = D_{1\,n}\ ,$$

$$A_{2m}\, U_{2\,m+1}^{j+1} + B_{2m}\, U_{2\,m}^{j+1} + C_{2m}\, U_{2\,m-1}^{j+1} = D_{2m}\ .$$

The expressions for the coefficients A, B, C, D are written by the known form. The passing ratio for the first trinominal equation is written as:

$$U_{1\,n+1} = K_{1\,n+1}\, U_{1\,n} + L_{1\,n}\ .$$

Using the boundary condition (7) in the difference form, it can be obtained the initial values for the passing coefficients:

$$K_{1\,N} = \frac{\cos\varphi_m - 1}{\cos\varphi_m - 1 + \Delta\mu_{N-1} Bi},\ \ L_{1\,N} = 0\,.$$

Further, all passing coefficients $K_{1\,n}, L_{1\,n}$ on the diminishing index values $n = N-1, N-2, \ldots, 3, 2.$ are determined according to the re-current formulas. At $n = 2$ a direct passing expression is written

$$U_{1\,2} = K_{1\,2}\, U_{1\,1} + L_{1\,2}\,. \tag{14}$$

Therefore, it is impossible yet to perform a direct passing for $U_{1\,n}$ determining. Because value $U_{1\,1}$ is unknown, it shall be determined from the conjugate conditions (12), but previously, the difference trinominal correlations are to be obtained for the thermal transfer in the pipeline, i.e. the difference scheme for the equation was constructed (11). To approximate the hyperbolic left section, a four-point "block" scheme was used, it is absolutely stable and of the second order of accuracy. An operator of the second order is written on a three-point template with the step $\Delta\rho_l = \rho_{l+1} - \rho_l$, where index l passes the values $l = 2, 3, \ldots, L-2, L-1$. With increasing of the radius ρ, increasing from the pipeline axis is observed, and the running correlation has the form:

$$\Theta_{l-1} = K_l\, \Theta_l + L_l\ .$$

From the symmetry conditions the initial values of the running coefficients $K_2 = 1$, $L_2 = 0.$ are determined on the pipe axis. At the upper bound (on the pipe wall) the correlation

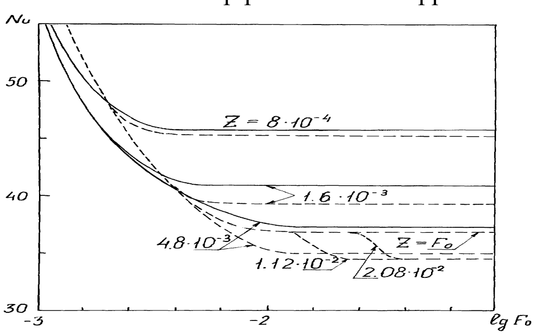

Fig. 1

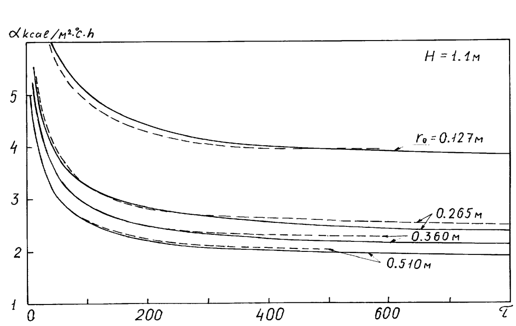

Fig. 2

$$\Theta_{L-1} = K_L \, \Theta_L + L_L \,, \tag{15}$$

is obtained which encloses the conjugate conditions. If to write the equality (12) in the form of a difference scheme, so four equations relative to four unknown ones are obtained, they are (12), (14), (15). Finally, the value of the soil temperature at the contact plane will take the form:

$$U_{11} = \frac{\Omega \, L_L + \Phi \, L_{12} + \Omega \, (K_L - 1)(1 - E)}{\Phi \, (1 - K_{12}) + \Omega \, (K_L - 1)} \,, \tag{16}$$

where

$$\Omega = - \; \lambda_{\text{liq}} \; T_e \; /(r_0 \;\; \Delta\rho_{L-1}), \; \Phi = \lambda_{\text{s}} \; t_{\text{air}}(H / r_0 \; - \; \cos\varphi)/(c \;\; \Delta\mu_1),$$

$$E = \delta \, t_{\text{air}} \; / \; T_e \; , \; \Theta_L = 1 \; - \; E(\; 1 \; + \; U_{11}).$$

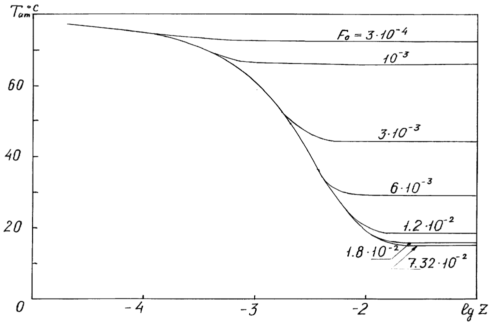

Fig. 3

Using U_{11} in (14) and etc., and Θ_L in (15), it can be determined a temperature profile in the pipeline and solution of the splitting equation (13) relatively to U_1. Further, the equation solution relatively to U_2 using a running along the coordinate lines of axis φ. are seen. The initial field is determined by solution of $U_{2\,n,m}^{j} = U_{1\,n,m}^{j+1}$, and as a result, the solution of the thermal conduction equation (6) in the soil is found using a longitudinal-transverse running $U_{n,m}^{j+1} = U_{2\,n,m}^{j+1}$.

The algorithms of nonstationary heat exchange were previously checked in independent calculations both for the soil with a isothermal pipe and for the turbulent flow regime in the pipe. The calculations were compared with results of [1, 3] and are presented in Figs. 1 and 2 .

Figure 1 shows a comparison of our numerical calculations (round points) with the calculation results of [3] under set conditions.

Figure 2 presents the Nusselt number $\text{Nu} \; (Z, \; F_0)$ at $\text{Re} = 10^4, \; \text{Pr} = 1,$ for the turbulent flow in the pipe and for uneven change of the heat flow $Q(F_0) = \text{const}$ on the pipe wall, our computations in comparison with calculations made in [1] are marked by a dashed line.

The calculations of the certain pipeline are presented in Fig. 3.

$$r_0 = 0.36 \;\; \text{m} \; , \; H = 1.1 \;\; \text{m} \; , \; \text{Re} = 10^{\,5}, \; \lambda_{\text{s}} = 1.28 \;\; \text{kcal} / \text{m} \cdot h \; ^{\text{o}}\,\text{C}, \; \text{Pr} = 1, \; P_{\;T} = 1,$$

$$a_{\,\text{s}} = 2.37 \;\; * \;\; 10^{-3} \;\; \text{m}^2 \, / \;\; h \; , \; \alpha_{\,\text{air}} = 12.8 \;\; \text{kcal} / \text{m} \cdot \text{h} \; ^{\circ}\,\text{C}, \; T_e = 80 \; ^{\text{o}}\,\text{C}, \; t \; = 15 \;\; ^{\text{o}}\text{C};$$

$$\lambda_{\;\text{ж}} = 3.2 \;\; {*}10^{\;-2} \;\; \text{kcal/м} \cdot \text{h} \;\; ^{\circ}\text{C} \; ;$$

In the Fig. 3 shows dependence of the mean mass temperature liquid T_{am} from distance and times.

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